Trigonometric Identities and Equations- MS

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\frac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\frac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to form a correct simplified equ in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is fo correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to fi least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \frac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

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2.

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30 \times 6.5)° = \text{awrt } 4.48\text{ m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

z

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt\ 4.48\text{m}$. The units must be seen somewhere in (a). So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k,\quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{invsin } \textit{their} - 0.6$ to give the first value of t where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe
Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe
DO NOT allow 646 minutes or 10 hours 46 minutes.

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3.

Question	Scheme	Marks	AOs
12(a)	$\frac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\frac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{3+2\cos\theta}$	M1	1.1b
	$\equiv\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$	A1	1.1b
	$\equiv\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$	M1	1.1b
	$\equiv 4-5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4+3\sin x=4-5\cos x\Rightarrow\tan x=-\frac{5}{3}$	M1	2.1
	$x=\text{awrt } 121°, 301°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

(a)

M1: Uses the identity $\sin^2\theta=1-\cos^2\theta$ within the fraction

A1: Correct (simplified) expression in just $\cos\theta$ $\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$ or exact equivalent such as $\frac{(3+2\cos\theta)(4-5\cos\theta)}{3+2\cos\theta}$ Allow for $\frac{12-7u-10u^2}{3+2u}$ where they introduce $u=\cos\theta$
We would condone mixed variables here.

M1: A correct attempt to factorise the numerator, usual rules. Allow candidates to use $u=\cos\theta$ oe

A1*: A fully correct proof with correct notation and no errors.
Only withhold the last mark for (1) Mixed variable e.g. θ and x's (2) Poor notation $\cos\theta^2\leftrightarrow\cos^2\theta$ or $\sin^2=1-\cos^2$ within the solution.
Don't penalise incomplete lines if it is obvious that it is just part of their working

E.g. $\frac{10\sin^2\theta-7\cos\theta+2}{3+2\cos\theta}\equiv\frac{10\left(1-\cos^2\theta\right)-7\cos\theta+2}{}=\frac{12-7\cos\theta-10\cos^2\theta}{3+2\cos\theta}$

(b)

M1: Attempts to use part (a) and proceeds to an equation of the form $\tan x = k, \quad k \neq 0$

Condone $\theta \leftrightarrow x$ Do not condone $a\tan x = 0 \Rightarrow \tan x = b \Rightarrow x = ...$

Alternatively squares $3\sin x = -5\cos x$ and uses $\sin^2 x = 1 - \cos^2 x$ oe to reach $\sin x = A, -1 < A < 1$ or $\cos x = B, -1 < B < 1$

A1: Either $x =$ awrt $121°$ **or** $301°$. Condone awrt 2.11 or 5.25 which are the radian solutions

A1: Both $x =$ awrt $121°$ **and** $301°$ and no other solutions.

Answers without working, or with no incorrect working in (b).
Question states hence or otherwise so allow
For 3 marks both $x =$ awrt $121°$ **and** $301°$ and no other solutions.
For 1 marks scored SC 100 for either $x =$ awrt $121°$ **or** $301°$

Alternative proof in part (a):

M1: Multiplies across and form 3TQ in $\cos\theta$ on rhs

$10\sin^2\theta - 7\cos\theta + 2 = (4 - 5\cos\theta)(3 + 2\cos\theta) \Rightarrow 10\sin^2\theta - 7\cos\theta + 2 = A\cos^2\theta + B\cos\theta + C$

A1: Correct identity formed $10\sin^2\theta - 7\cos\theta + 2 = -10\cos^2\theta - 7\cos\theta + 12$

dM1: Uses $\cos^2\theta = 1 - \sin^2\theta$ on the rhs or $\sin^2\theta = 1 - \cos^2\theta$ on the lhs

Alternatively proceeds to $10\sin^2\theta + 10\cos^2\theta = 10$ and makes a statement about $\sin^2\theta + \cos^2\theta = 1$ oe

A1*: Shows that $(4 - 5\cos\theta)(3 + 2\cos\theta) \equiv 10\sin^2\theta - 7\cos\theta + 2$ oe AND makes a minimal statement "hence true"

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4.

Question	Scheme	Marks	AOs
12 (a)	$4\cos\theta - 1 = 2\sin\theta\tan\theta \Rightarrow 4\cos\theta - 1 = 2\sin\theta \times \frac{\sin\theta}{\cos\theta}$	M1	1.2
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe	A1	1.1b
	$\Rightarrow 4\cos^2\theta - \cos\theta = 2\left(1-\cos^2\theta\right)$	M1	1.1b
	$6\cos^2\theta - \cos\theta - 2 = 0$ *	A1*	2.1
		(4)	
(b)	For attempting to solve given quadratic	M1	1.1b
	$(\cos 3x) = \frac{2}{3}, -\frac{1}{2}$	B1	1.1b
	$x = \frac{1}{3}\arccos\left(\frac{2}{3}\right)$ or $\frac{1}{3}\arccos\left(-\frac{1}{2}\right)$	M1	1.1b
	$x = 40°, 80°$, awrt $16.1°$	A1	2.2a
		(4)	
			(8 marks)

(a)

M1: Recall and use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ Note that it cannot just be stated.

A1: $4\cos^2\theta - \cos\theta = 2\sin^2\theta$ oe.

This is scored for a correct line that does not contain any fractional terms.

It may be awarded later in the solution after the identity $1-\cos^2\theta = \sin^2\theta$ has been used Eg for $\cos\theta(4\cos\theta - 1) = 2\left(1-\cos^2\theta\right)$ or equivalent

M1: Attempts to use the correct identity $1-\cos^2\theta = \sin^2\theta$ to form an equation in just $\cos\theta$

A1*: Proceeds to correct answer through rigorous and clear reasoning. No errors in notation or bracketing. For example $\sin^2\theta = \sin\theta^2$ is an error in notation

(b)

M1: For attempting to solve the given quadratic "$6y^2 - y - 2 = 0$ " where y could be $\cos 3x$, $\cos x$, or even just y. When factorising look for $(ay+b)(cy+d)$ where $ac = \pm 6$ and $bd = \pm 2$

This may be implied by the correct roots (even award for $\left(y \pm \frac{2}{3}\right)\left(y \pm \frac{1}{2}\right)$), an attempt at factorising, an attempt at the quadratic formula, an attempt at completing the square and even $\pm$ the correct roots.

B1: For the roots $\frac{2}{3}, -\frac{1}{2}$ oe

M1: Finds at least one solution for x from $\cos 3x$ **within the given range** for their $\frac{2}{3}, -\frac{1}{2}$

A1: $x = 40°, 80°$, awrt $16.1°$ **only** Withhold this mark if there are **any** other values even if they are outside the range. Condone 40 and 80 appearing as 40.0 and 80.0

May 2017 Mathematics Advanced Paper 1: Pure Mathematics 2

5.

Question Number	Scheme		Marks
8. (a)	Way 1 $1-\sin^2 x = 8\sin^2 x - 6\sin x$	Way 2 $2=(3\sin x-1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$	B1
	E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$	so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$	M1
	So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2 = (3\sin x - 1)^2$ *	$8\sin^2 x - 6\sin x = \cos^2 x$ *	A1cso*
			(3)
(b)	Way 1: $(3\sin x - 1) = (\pm)\sqrt{2}$	Way 2: Expands $(3\sin x - 1)^2 = 2$ and uses quadratic formula on 3TQ	M1
	$\sin x = \dfrac{1\pm\sqrt{2}}{3}$ or awrt 0.8047 **and** awrt − 0.1381		A1
	$x = 53.58$, 126.42 (or 126.41), 352.06, 187.94		dM1A1 A1
			(5) **[8]**

	Notes
(a)	**Way 1** B1: Uses $\cos^2 x = 1-\sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2=(3\sin x-1)^2$. If sin is used throughout instead of sin*x* it is A0. **Way 2** B1: **Needs correct expansion** and **split** M1: Collects $1-\sin^2 x$ together A1*: Conclusion and no errors seen
(b)	M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0. A1: **Both** correct answers for sin*x* (need plus and minus). Need not be simplified. dM1: Uses inverse sin to give one of the given correct answers 1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**:– in the final three marks lose first A1 then ft other angles for second A mark) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 2

6.

Question Number	Scheme		Marks
6.	$1-2\cos\left(\theta-\frac{\pi}{5}\right)=0\,;\ -\pi<\theta\leqslant\pi$		
(i)	$\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$	Rearranges to give $\cos\left(\theta-\frac{\pi}{5}\right)=\frac{1}{2}$ or $-\frac{1}{2}$	M1
	$\theta=\left\{-\frac{2\pi}{15},\ \frac{8\pi}{15}\right\}$	At least one of $-\frac{2\pi}{15}$ or $\frac{8\pi}{15}$ or -24° or 96° or awrt 1.68 or awrt -0.419	A1
		Both $-\frac{2\pi}{15}$ and $\frac{8\pi}{15}$	A1
			[3]
NB Misread	**Misreading** $\frac{\pi}{5}$ as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark		
	$4\cos^2 x+7\sin x-2=0,\ 0\leqslant x<360^{\circ}$		
(ii)	$4(1-\sin^2 x)+7\sin x-2=0$	Applies $\cos^2 x=1-\sin^2 x$	M1
	$4-4\sin^2 x+7\sin x-2=0$		
	$4\sin^2 x-7\sin x-2=0$	Correct 3 term, $4\sin^2 x-7\sin x-2\ \{=0\}$	A1 oe
	$(4\sin x+1)(\sin x-2)\ \{=0\}\ ,\ \sin x=\ldots$	Valid attempt at solving and $\sin x=\ldots$	M1
	$\sin x=-\frac{1}{4},\ \ \{\sin x=2\}$	$\sin x=-\frac{1}{4}$ (See notes.)	A1 cso
	$x=\text{awrt}\ \{194.5,\ 345.5\}$	At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
		awrt 194.5 **and** awrt 345.5	A1
			[6]
			9

NB Misread	**Writing equation as** $4\cos^2 x-7\sin x-2=0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6		
	$4(1-\sin^2 x)-7\sin x-2=0$		**M1**
	$4\sin^2 x+7\sin x-2=0$		**A0**
	$(4\sin x-1)(\sin x+2)\ \{=0\}\ ,\ \sin x=\ldots$	Valid attempt at solving and $\sin x=\ldots$	**M1**
	$\sin x=+\frac{1}{4},\ \ \{\sin x=-2\}$	$\sin x=\frac{1}{4}$ (See notes.)	**A0**
	$x=\text{awrt}165.5$		**A1ft**
	Incorrect answers		**A0**

Question 6 Notes

(i)	**M1**	Rearranges to give $\cos\left(\theta-\frac{\pi}{5}\right)=\pm\frac{1}{2}$
	Note	M1 can be implied by seeing either $\frac{\pi}{3}$ or 60° as a result of taking $\cos^{-1}(\ldots)$.
	A1	Answers **may be in degrees or radians** for this mark and may have just one correct answer Ignore mixed units in working if correct answers follow (recovery)
	A1	Both answers correct and in radians as multiples of π $\ -\frac{2\pi}{15}$ and $\frac{8\pi}{15}$ Ignore EXTRA solutions outside the range $-\pi<\theta\leq\pi$ but lose this mark for extra solutions in this range.

(ii)	**1st M1**	Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.]
	1st A1	Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2 \{= 0\}$ or $-4\sin^2 x + 7\sin x + 2 \{= 0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc.
	2nd M1	For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, s, y, x or $\sin x$, and an attempt to find at least one of the solutions for sin$x$. This solution may be outside the range for sin$x$
	2nd A1	$\sin x = -\frac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$.
	Note	$\sin x = -\frac{1}{4}$ can be implied by later correct working if no errors are seen.
	3rd A1ft	At least one of awrt 194.5 **or** awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through. Only follow through on the error $\sin x = \frac{1}{4}$ and allow for 165.5 special case (as this is equivalent work) This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip.
	4th A1	awrt 194.5 **and** awrt 345.5
	Note	If there are any EXTRA solutions inside the range $0 \leq x < 360°$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark. Ignore EXTRA solutions outside the range $0 \leq x < 360°$.
	Special Cases	Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error) Answers in radians:– **lose final** mark so either or both of 3.4, 6.0 gets A1ftA0 It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in sin$x$ = -1/4 then correct work follows.

May 2015 Mathematics Advanced Paper 1: Pure Mathematics 2

7.

Question Number	Scheme	Marks
8. (i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$ Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)	M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)	A1 **(3)**
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$	dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	$x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1 **(3)** **[9]**

Notes

(i) **M1**: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark)

Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

A1: Need all three correct answers in terms of π and **no extras in range**.

Three correct answers implies M1M1A1

NB : $\theta = 20°, 80°, 140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

(ii) (a) **M1:** Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$).

This must be awarded in (ii) (a) for an expression with k not after $k = 3$ is substituted.

dM1: Uses formula or completion of square to obtain $\cos x =$ expression in k

(Factorisation attempt is M0) **A1:** cao - award for their final simplified expression

(b) **M1**: **Either** attempts to substitute $k = 3$ into their answer to obtain two values for $\cos x$

Or restarts with $k = 3$ to find two values for $\cos x$ (They cannot earn marks in ii(a) for this)

In both cases they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for $\cos x$ may be >1 or < -1

dM1: Obtains **two correct** values for x

A1: Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

May 2013 Mathematics Advanced Paper 1: Pure Mathematics 2

8.

Question Number	Scheme	Marks
8. (i)	$(\lvert\alpha\rvert = 56.3099...)$	
	$x = \{\alpha + 40 = 96.309993...\} =$ **awrt 96.3**	B1
	$x - 40^{\circ} = -180 + "56.3099"...$ or $x - 40^{\circ} = -\pi + "0.983"...$	M1
	$x = \{-180 + 56.3099... + 40 = -83.6901...\} =$ **awrt -83.7**	A1
		(3)
(ii)(a)	$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	M1
	$\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	dM1
	$1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ *	A1 cso *
		(3)
(b)	$\cos\theta = \dfrac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	
	or $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0, q \neq 0$ so $\cos\theta = ...$	M1
	One solution is 72° or 144°, Two solutions are 72° and 144°	A1, A1
	$\theta = \{72, 144, 216, 288\}$	M1 A1
		(5) **[11]**

	Notes for Question 8
(i)	B1: 96.3 by any or no method M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360). A1: awrt –83.7 Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned) Working in radians – could earn M1 for $x - 40^{\circ} = -\pi + "0.983"...$ so B0M1A0
(ii) (a)	M1: uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \dfrac{\sin}{\cos}$, with no argument) dM1: uses $\sin^2\theta = 1 - \cos^2\theta$ (quoted correctly) in equation A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0"
(b)	M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0. 1st A1: Either 72 **or** 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.) M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M) A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5

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9.

Question Number	Scheme		Marks
4.			
	$\cos^{-1}(-0.4) = 113.58\ (\alpha)$	Awrt 114	B1
	$3x-10=\alpha \Rightarrow x=\dfrac{\alpha+10}{3}$	Uses their α to find x. Allow $x=\dfrac{\alpha\pm10}{3}$ **not** $\dfrac{\alpha}{3}\pm10$	M1
	Note: If $x=\dfrac{\alpha\pm10}{3}$ is not clearly applied from their first angle it may be recovered if applied to their second or third angle.		
	$x=41.2$	Awrt	A1
	$(3x-10=)360-\alpha\ \ (246.4....)$	$360-\alpha$ (can be implied by 246.4...)	M1
	$x=85.5$	Awrt	A1
	$(3x-10=)360+\alpha\ (=473.57....)$	$360+\alpha$ (Can be implied by 473.57...)	M1
	$x=161.2$	Awrt	A1
	Note 1: Do not penalise incorrect accuracy more than once and penalise it the first time it occurs. E.g if answers are only given to the nearest integer (41, 85, 161) only the first A mark that would otherwise be scored is lost.		
	Note 2: Ignore any answers outside the range. For extra answers in range in an otherwise **fully** correct solution lose final A1		
	Note 3: Lack of working means that it is sometimes not clear where their intermediate angles are coming from. In these cases, if the final answers are incorrect score M0.		
	Note 4: Candidates are unlikely to be working in radians <u>deliberately</u> but may have their calculator in radian mode (gives α = 1.98). In such cases the main scheme should be applied and the method marks are available. If you suspect that the candidate is working in radians correctly then please use the review mechanism and/or consult your team leader.		
Way 2	$\cos^{-1}(0.4) = 66.42\ (\alpha)$		
	$180-66.42=113.58$	Awrt 114	B1
	$3x-10=113.58 \Rightarrow x=\dfrac{113.58+10}{3}$	Uses their 113.58 to find x	M1
	$x=41.2$	Awrt	A1
	$3x-10=180+\alpha\ \ (246.4....)$	$180+\alpha$	M1
	to give $x=85.5$		A1
	$3x-10=540-\alpha\ \ (473.57....)$	$540-\alpha$	M1
	to give $x=161.2$		A1
	Special case - takes 0.4 as -0.4		
	$\cos^{-1}(0.4) = 66.42\ (\alpha)$		B0
	$3x-10=66.4 \Rightarrow x=\dfrac{66.4\pm10}{3}$		M1
	$x=41.2$		A0
	$3x-10=360-\alpha\ \ (293.6....)$		M1
	$x=101.2$		A0
	$3x-10=360+\alpha\ \ (426.4....)$		M1
	$x=145.5$		A0
			(3/7)

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10.

Question number	Scheme		Marks
6(a)	States or uses $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$		M1
	$\dfrac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1-5\cos 2x) = 0$ *		A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) **or** $2x = 281.54$ (or 281.6)		M1
	$x = 39.2$ (or 39.3), 140.8 (or 141)		A1, A1 (5)
			7 marks

Notes	(a) **M1**: Statement that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$. **A1**: the answer is given so all steps should be given. N.B. $\sin 2x - 5\sin 2x\cos 2x = 0$ or $-5\sin 2x\cos 2x + \sin 2x = 0$ or $\sin 2x(\dfrac{1}{\cos 2x} - 5) = 0$ o.e. **must be seen** and be followed by printed answer for A1 mark $\sin 2x = 5\sin 2x\cos 2x$ is not sufficient. (b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions **M1**: This mark for one of the two statements given (must relate to $2x$ not just to x) **A1, A1**: first A1 for 39.2, second for 140.8 *Special case* solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141 Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14) Excess answers **in range** lose last A1 Ignore excess answers outside range. All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here

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11.

Question number	Scheme	Marks
9 (i)	$\sin(3x-15)=\frac{1}{2}$ so $3x-15=30$ (α) and $x=15$	M1 A1
	Need $3x-15=180-\alpha$ or $3x-15=540-\alpha$	M1
	Need $3x-15=180-\alpha$ and $3x-15=360+\alpha$ and $3x-15=540-\alpha$	M1
	$x=55$ or 175	A1
	$x=$ 55 , 135 , 175	A1 (6)
Notes	**M1** Correct order of operation: inverse sine then linear algebra - not just $3x$-15 = 30 (slips in linear algebra lose Accuracy mark) **A1** Obtains first solution 15 **M1** Uses either 180 – α or 540 – α, **M1** uses all three 180 – α and 360 + α and 540 – α **A1**, for one further correct solution 55 or 175, (depends only on second M1) **A1** – all 3 further correct solutions If more than 4 solutions in range, lose last **A1** Common slips: Just obtains 15 and 55, or 15 and 175 – usually M1A1M1M0A1A0 Just obtains 15 and 135 is usually M1A1M0M0A0A0 (It is easy to get this erroneously) Obtains 5, 45, 125 and 165 – usually M1A0M1M1A0A0 Obtains 25, 65, 145, (185) usually M1A0M1M1A0A0 Working in radians – lose last A1 earned for $\frac{\pi}{12}, \frac{11\pi}{36}, \frac{3\pi}{4}$ and $\frac{35\pi}{36}$ or numerical equivalents Mixed radians and degrees is usually Method marks only Methods involving no working should be sent to Review	
9 (ii)	At least one of $(\frac{a\pi}{10}-b)=0$ (or $n\pi$) $(\frac{a3\pi}{5}-b)=\pi$ {or $(n+1)\pi$} or in degrees or $(\frac{a11\pi}{10}-b)=2\pi$ {or $(n+2)\pi$}	M1
	If two of **above equations** used eliminates a or b to find one or both of these or uses period property of curve to find a or uses other valid method to find either a or b (May see $\frac{5\pi}{10}a=\pi$ so $a=$)	M1
	Obtains $a=2$	A1
	Obtains $b=\frac{\pi}{5}$ (must be in radians)	A1 (4)

Notes	M1: Award for $(\frac{a\pi}{10}-b)=0$ or $\frac{a\pi}{10}=b$ BUT $\sin(\frac{a\pi}{10}-b)=0$ is M0 M1: As described above but solving $(\frac{a\pi}{10}-b)=0$ with $(\frac{a3\pi}{5}-b)=0$ is M0 (It gives $a=b=0$) Special cases: Can obtain full marks here for both correct answers with no working M1M1A1A1 For $a=2$ only, with no working, award M0M1A1A0 For $b=\frac{\pi}{5}$ only with no working M1M0A0A1
Alternative	Some use translations and stretches to give answers. If they achieve $a=2$ they earn second method and first accuracy. If they achieve correct value for b they earn first method and second accuracy. Common error is $a=2$ and $b=\frac{\pi}{10}$. This is usually M0M1A1A0 unless they have stated $(\frac{a\pi}{10}-b)=0$ earlier in which case they earn first M1.

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12.

7.	(a) $3\sin(x+45^\circ)=2$; $0\le x<360^\circ$ (b) $2\sin^2 x+2=7\cos x$; $0\le x<2\pi$		
(a)	$\sin(x+45^\circ)=\frac{2}{3}$, so $(x+45^\circ)=41.8103...$ $(\alpha=41.8103...)$	$\sin^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt 0.73^c	M1
	So, $x+45^\circ=\{138.1897...,\ 401.8103...\}$	$x+45^\circ$ = either "$180-$ their α" or "$360^\circ+$ their α" (α could be in radians).	M1
	and $x=\{93.1897...,\ 356.8103...\}$	Either awrt 93.2° or awrt 356.8°	A1
		Both awrt 93.2° and awrt 356.8°	A1
			[4]
(b)	$2(1-\cos^2 x)+2=7\cos x$	Applies $\sin^2 x=1-\cos^2 x$	M1
	$2\cos^2 x+7\cos x-4=0$	Correct 3 term, $2\cos^2 x+7\cos x-4\ \{=0\}$	A1 **oe**
	$(2\cos x-1)(\cos x+4)\ \{=0\}$, $\cos x=...$	Valid attempt at solving and $\cos x=...$	M1
	$\cos x=\frac{1}{2}$, $\{\cos x=-4\}$	$\cos x=\frac{1}{2}$ (See notes.)	A1 cso
	$\left(\beta=\frac{\pi}{3}\right)$		
	$x=\frac{\pi}{3}$ or $1.04719...^c$	Either $\frac{\pi}{3}$ or awrt 1.05^c	B1
	$x=\frac{5\pi}{3}$ or $5.23598...^c$	Either $\frac{5\pi}{3}$ or awrt 5.24^c or $2\pi-$ their β (See notes.)	B1 ft
			[6] **10**

Question Number	Scheme	Marks
(a)	1^{st} M1: can also be implied for $x = $ awrt -3.2 2^{nd} M1: for $x + 45^{\circ}$ = either "180 – their α" or "360° + their α". This can be implied by later working. The candidate's α could also be in radians. **Note that this mark is not** for $x =$ either "180 – their α" or "360° + their α". **<u>Note:</u>** Imply the first two method marks or award M1M1A1 for either awrt 93.2° or awrt 356.8°. **<u>Note:</u>** Candidates who apply the following incorrect working of $3\sin(x + 45^{\circ}) = 2$ $\Rightarrow 3(\sin x + \sin 45) = 2$, etc will usually score M0M0A0A0. If there are any EXTRA solutions inside the range $0 \le x < 360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360$. **Working in Radians:** Note the answers in radians are $x =$ awrt 1.6, awrt 6.2 If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) **No working:** Award M1M1A1A0 for one of awrt 93.2° or awrt 356.8° seen without any working. Award M1M1A1A1 for both awrt 93.2° and awrt 356.8° seen without any working. Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x$ {and not x} = {awrt 93.2, awrt 356.8}	

Question Number	Scheme	Marks
(b)	1^{st} M1: for a correct method to use $\sin^2 x = 1 - \cos^2 x$ on the given equation. Give bod if the candidate omits the bracket when substituting for $\sin^2 x$, but $2 - \cos^2 x + 2 = 7\cos x$, without supporting working, (eg. seeing "$\sin^2 x = 1 - \cos^2 x$") would score 1^{st} M0. Note that applying $\sin^2 x = \cos^2 x - 1$, scores M0. 1^{st} A1: for obtaining either $2\cos^2 x + 7\cos x - 4$ or $-2\cos^2 x - 7\cos x + 4$. 1^{st} A1: can also awarded for a correct three term equation eg. $2\cos^2 x + 7\cos x = 4$ or $2\cos^2 x = 4 - 7\cos x$ etc. 2^{nd} M1: for a valid attempt at factorisation of a quadratic (either 2TQ or 3TQ) in cos, can use any variable here, c, y, x or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 2^{nd} A1: for $\cos x = \frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x = -4$, but penalise if candidate states an incorrect result e.g. $\cos x = 4$. If they have used a substitution, a correct value of their c or their y or their x. **Note:** 2^{nd} A1 for $\cos x = \frac{1}{2}$ can be implied by later working. 1^{st} B1: for either $\frac{\pi}{3}$ or awrt 1.05^{c}	

	2^{nd} B1: for either $\frac{5\pi}{3}$ or awrt 5.24^c or can be ft from 2π – their β or 360° – their β where $\beta = \cos^{-1}(k)$, such that $0 < k < 1$ or $-1 < k < 0$, but $k \neq 0$, $k \neq 1$ or $k \neq -1$. If there are any EXTRA solutions inside the range $0 \le x < 2\pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$. **Working in Degrees:** Note the answers in degrees are $x = 60,\ 300$ If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.) **Answers from no working:** $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ scores M0A0M0A0B1B1, $x = 60$ and $x = 300$ scores M0A0M0A0B1B0, $x = \frac{\pi}{3}$ ONLY or $x = 60$ ONLY scores M0A0M0A0B1B0, $x = \frac{5\pi}{3}$ ONLY or $x = 120$ ONLY scores M0A0M0A0B0B1. **No working:** You cannot apply the ft in the B1ft if the answers are given with NO working. Eg: $x = \frac{\pi}{5}$ and $x = \frac{9\pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0. **For candidates using trial & improvement, please forward these to your Team Leader.**

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13.

Question Number	Scheme		Marks
7.			
(a)	$3\sin^2 x + 7\sin x = \cos^2 x - 4;\ \ 0 \le x < 360^\circ$		
	$3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$		M1
	$4\sin^2 x + 7\sin x + 3 = 0$ **AG**		A1 * cso
			(2)
(b)	$(4\sin x + 3)(\sin x + 1)\ \{= 0\}$	Valid attempt at factorisation and $\sin x = ...$	M1
	$\sin x = -\frac{3}{4},\quad \sin x = -1$	Both $\sin x = -\frac{3}{4}$ and $\sin x = -1$.	A1
	$(\lvert\alpha\rvert = 48.59...)$		
	$x = 180 + 48.59$ or $x = 360 - 48.59$	Either $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$	dM1
	$x = 228.59...,\ x = 311.41...$	Both awrt 228.6 and awrt 311.4	A1
	$\{\sin x = -1\} \Rightarrow x = 270$	270	B1
			(5) **[7]**
	<u>Notes</u>		
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$). Note that applying $\cos^2 x = \sin^2 x - 1$, scores M0. A1 for obtaining the printed answer without error (except for implied use of zero.), although the equation at the end of the proof **must be = 0.** Solution **just** written only as above would score M1A1.		

(b)	1^{st} M1 for a valid attempt at factorisation, can use any variable here, s, y, x or $\sin x$, and an attempt to find at least one of the solutions. *Alternatively*, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution. 1^{st} A1 for the two correct values of $\sin x$. If they have used a substitution, a correct value of their s or their y or their x. 2^{nd} M1 for solving $\sin x = -k$, $0 < k < 1$ and realising a solution is either of the form $(180 + \lvert\alpha\rvert)$ or $(360 - \lvert\alpha\rvert)$ where $\alpha = \sin^{-1}(k)$. Note that you **cannot** access this mark from $\sin x = -1 \Rightarrow x = 270$. Note that this mark is dependent upon the 1^{st} M1 mark awarded. 2^{nd} A1 for both awrt 228.6 and awrt 311.4 B1 for 270. If there are any EXTRA solutions inside the range $0 \le x < 360^\circ$ and the candidate would otherwise score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \le x < 360^\circ$. **Working in Radians:** Note the answers in radians are $x = 3.9896..., 5.4351..., 4.7123...$ If a candidate works in radians then mark part (b) as above awarding the 2^{nd} A1 for both awrt 4.0 and awrt 5.4 and the B1 for awrt 4.7 or $\frac{3\pi}{2}$. If the candidate would then score FULL MARKS then withhold the final bA2 mark (the fourth mark in this part of the question.) **No working:** Award B1 for 270 seen without any working. Award M0A0M1A1 for awrt 228.6 and awrt 311.4 seen without any working. Award M0A0M1A0 for any one of awrt 228.6 or awrt 311.4 seen without any working.

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14.

Question Number	Scheme	Marks	
5	(a) $\tan\theta = \frac{2}{5}$ (or 0.4) (i.s.w. if a value of θ is subsequently found) Requires the correct value with no incorrect working seen.	B1	(1)
	(b) awrt 21.8 (α) (Also allow awrt 68.2, ft from $\tan\theta = \frac{5}{2}$ in (a), but no other ft) (This value must be seen in part (b). It may be implied by a correct solution, e.g. 10.9)	B1	
	$180 + \alpha$ $(= 201.8)$, or $90 + (\alpha/2)$ (if division by 2 has already occurred) (α found from $\tan 2x = ...$ or $\tan x = ...$ or $\sin 2x = \pm...$ or $\cos 2x = \pm...$)	M1	
	$360 + \alpha$ $(= 381.8)$, or $180 + (\alpha/2)$ (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$) OR $540 + \alpha$ $(= 561.8)$, or $270 + (\alpha/2)$ (α found from $\tan 2x = ...$)	M1	
	Dividing at least one of the angles by 2 (α found from $\tan 2x = ...$ or $\sin 2x = ...$ or $\cos 2x = ...$)	M1	
	$x = 10.9,\ 100.9,\ 190.9,\ 280.9$ (Allow awrt)	A1	(5) 6

(b) Extra solution(s) in range: Loses the final A mark.

Extra solutions outside range: Ignore (whether correct or not).

Common answers:

10.9 and 100.9 would score B1 M1 M0 M1 A0 (Ensure that these M marks are awarded)

10.9 and 190.9 would score B1 M0 M1 M1 A0 (Ensure that these M marks are awarded)

Alternatives:

(i) $2\cos 2x - 5\sin 2x = 0$ $R\cos(2x+\lambda)=0$ $\lambda = 68.2 \Rightarrow 2x + 68.2 = 90$ B1

$2x+\lambda = 270$ M1

$2x+\lambda = 450$ or $2x+\lambda = 630$ M1

Subtracting λ and dividing by 2 (at least once) M1

(ii) $25\sin^2 2x = 4\cos^2 2x = 4(1-\sin^2 2x)$

$29\sin^2 2x = 4$ $2x = 21.8$ B1

The M marks are scored as in the main scheme, but extra solutions will be likely, losing the A mark.

Using radians:

B1: Can be given for awrt 0.38 (β)

M1: For $\pi+\beta$ or $180+\beta$

M1: For $2\pi+\beta$ or $3\pi+\beta$ (Must now be consistently radians)

M1: For dividing at least one of the angles by 2

A1: For this mark, the answers must be in degrees.

(Correct) answers only (or by graphical methods):

B and M marks can be awarded by implication, e.g.

10.9 scores B1 M0 M0 M1 A0

10.9, 100.9 scores B1 M1 M0 M1 A0

10.9, 100.9, 190.9, 280.9 scores full marks.

Using 11, etc. instead of 10.9 can still score the M marks by implication.

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15.

Question Number	Scheme	Marks
Q2 (a)	$5\sin x = 1 + 2\left(1-\sin^2 x\right)$	M1
	$2\sin^2 x + 5\sin x - 3 = 0$ **(*)**	A1cso (2)
(b)	$(2s-1)(s+3)=0$ **giving** s =	M1
	$[\sin x = -3$ has no solution$]$ **so** $\sin x = \frac{1}{2}$	A1
	$\therefore\ x = 30,\ 150$	B1, B1ft (4) **[6]**
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1-\sin^2 x$) A1 need 3 term quadratic printed in any order with =0 included	

(b)	M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or $\sin x$) A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ **or** $x = \sin^{-1}\frac{1}{2}$ $y = \frac{1}{2}$ is A0 (unless followed by $x = 30$) B1 for 30 (α) not dependent on method 2nd B1 for 180 - α provided in required range (otherwise 540 - α) <u>Extra solutions outside required range</u>: Ignore <u>Extra solutions inside required range</u>: Lose final B1 <u>Answers in radians</u>: Lose final B1 S.C. Merely writes down two correct answers is M0A0B1B1 Or $\sin x = \frac{1}{2}$ $\therefore$ $x = 30,\ 150$ **is M1A1B1B1** Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1 **NB** Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$ $[\sin x = 3 \text{ gives no solution}]$ $\sin x = -\frac{1}{2}$ $\Rightarrow$ $x = 210, 330$ This earns M1 A0 B0 B1ft Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30°, 150°$ This would be M1 A0 B1 B1	

June 2013 Mathematics Advanced Paper 1: Pure Mathematics 3

16.

Question Number	Scheme	Marks
3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x° = \frac{1}{3}\tan 40°$ * cao	A1 *
		(4)
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$	M1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	A1
		(4)
		[8 marks]

Alt 1 **3(a)**	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ $\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	M1 M1 A1,A1
Alt 2 **3(a)**	$2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$ $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ $\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	 M1 M1 A1,A1

Notes for Question 3	
(a)	
M1	Expand both expressions using $\cos(x+50) = \cos x\cos 50 - \sin x\sin 50$ and $\sin(x+40) = \sin x\cos 40 + \cos x\sin 40$. Condone a missing bracket on the lhs. The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions. Allow if written separately and not in a connected equation.
M1	Divide by $\cos x$ to reach an equation in $\tan x$. Below is an example of M1M1 with incorrect sign on left hand side $2\cos x\cos 50 + 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ $\Rightarrow 2\cos 50 + 2\tan x\sin 50 = \tan x\cos 40 + \sin 40$ This is independent of the first mark.
A1	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$ Accept for this mark $\tan x =$ awrt 0.28... as long as M1M1 has been achieved.
A1*	States or uses cos50=sin40 and cos40=sin50 leading to showing $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \dfrac{\sin 40}{3\cos 40} = \dfrac{1}{3}\tan 40$ This is a given answer and all steps above must be shown. The line above is acceptable. Do not allow from $\tan x =$ awrt 0.28...

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3}\tan 40$

A1 Solves to find one solution of θ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of θ. It must be a full method but can be implied by any correct answer.

Accept $\theta = \dfrac{180 + their\alpha}{2}, (or) \dfrac{360 + their\alpha}{2}, (or) \dfrac{540 + their\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.
Ignore any extra solutions outside the range.
Withhold this mark for extras inside the range.
Condone a different variable. Accept x= 7.8, 97.8, 187.8, 277.8

Answers fully given in radians, loses the first A mark.
Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85
Mixed units can only score the first M 1

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17.

Question Number	Scheme		Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2(1 - 2\sin^2\theta) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$		
	$\theta = \{54, 126, 198, 342\}$	Any one correct answer 180-their pv All four solutions correct.	A1 dM1(*) A1
			[6]

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18.

Question Number	Scheme	Marks
1. (a)	$\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$	M1
	$\dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos\theta\cancel{\cos\theta}} = \tan\theta$ (as required) **AG**	A1 cso
		(2)
(b)	$2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$	M1
	θ_1 = awrt 26.6°	A1
	θ_2 = awrt -153.4°	A1$\sqrt{}$
		(3) **[5]**
	(a) M1: Uses **both** a correct identity for $\sin 2\theta$ **and** a correct identity for $\cos 2\theta$. Also allow a candidate writing $1 + \cos 2\theta = 2\cos^2\theta$ on the denominator. Also note that angles **must be consistent** in when candidates apply these identities. A1: Correct proof. No errors seen. (b) 1[st] M1 for either $2\tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$, seen or implied. A1: awrt 26.6 A1$\sqrt{}$: awrt -153.4° or $\theta_2 = -180^\circ + \theta_1$ **Special Case**: For candidate solving, $\tan\theta = k$, where $k \neq \frac{1}{2}$, to give θ_1 and $\theta_2 = -180^\circ + \theta_1$, then award M0A0B1 in part (b). **Special Case:** Note that those candidates who writes $\tan\theta = 1$, and gives ONLY two answers of 45° and -135° that are inside the range will be awarded SC M0A0B1.	